# SAT Math: The Hardest Question Types

A focused study sheet for a repeat test-taker. These are the question types that most often separate a 600s math score from a 700+ score. Each section gives the recognition cue, the method, a worked example, and the trap the College Board is counting on you to fall into.

### How to use this sheet

1. Read the **Recognize it** line first. Most lost points on hard questions come from not knowing which tool to reach for, not from weak algebra.
2. Cover the worked example and try it yourself before reading the solution.
3. Memorize the **Trap** lines. On your fourth attempt, you already know the math; the traps are what is still costing you.

## 1. Quadratics: Discriminant, Vertex, and Roots

**Recognize it:** The equation has an $x^2$ term and the question asks about *how many* solutions, the *minimum/maximum*, or the *sum/product* of solutions.

**Method:**
**Discriminant** of $ax^2 + bx + c = 0$ is $b^2 - 4ac$.
$> 0$ → two real solutions | $= 0$ → exactly one | $< 0$ → no real solutions.
**Vertex** is at $x = -b/(2a)$. Plug that x back in to get the min ($a > 0$) or max ($a < 0$).
**Sum of roots** $= -b/a$. **Product of roots** $= c/a$. (No factoring needed!)

**Worked example:**
For what value of $k > 0$ does $2x^2 + kx + 8 = 0$ have exactly one solution?
$b^2 - 4ac = 0 \rightarrow k^2 - 4(2)(8) = 0 \rightarrow k^2 = 64 \rightarrow k = 8$.

⚠ **Trap:** 'Exactly one solution' does NOT mean 'the solution is 1.' It means the discriminant is zero. Also: the minimum *value* is the y of the vertex, not the x.

## 2. Systems of Equations With No Solution or Infinitely Many

**Recognize it:** Two linear equations, one contains an unknown constant (k, a, c), and the question says 'no solution' or 'infinitely many solutions.'

**Method:**
Rewrite both as $Ax + By = C$.
**No solution** → same slope, different intercept → x- and y-coefficients are proportional but the constants are not.
**Infinitely many** → the equations are the same line → everything is proportional (one is a multiple of the other).
**Exactly one** → slopes differ.

**Worked example:**
$3x - 5y = 7$ and $6x + ky = 20$ have no solution. Find k.
Multiply the first by 2: $6x - 10y = 14$. For no solution, the left sides must match but the right sides differ. $k = -10$ (and $14 \neq 20$, so it is not the same line). ✓

⚠ **Trap:** Students set the whole equations equal and get a mess. Only the coefficients need to match. And check the constant: if it also matches, you have infinitely many, not zero.

## 3. Exponential Growth, Decay, and Rewriting Exponents

**Recognize it:** 'Increases by 12% each year,' 'halves every 5 hours,' or an expression like $16^{x/2}$ that must be rewritten with a different base.

**Method:**
**Model:** $y = a(1 + r)^t$ for growth, $a(1 - r)^t$ for decay. a = starting amount, r = rate as a decimal. If the change happens every n units of time, the exponent is t/n.
**Exponent rules:** $x^m \cdot x^n = x^{m+n}$; $(x^m)^n = x^{mn}$; $x^{-n} = 1/x^n$; $x^{1/n} = \sqrt[n]{x}$. To solve, rewrite both sides with the SAME base.

**Worked example:**
A sample of 500 grams decays by 20% every 3 days. Amount after d days?
$500(0.80)^{d/3}$. Check: at d = 3 the exponent is 1 → 400 g. ✓
Solve $4^{x+1} = 8^{x-1}$: $2^{2(x+1)} = 2^{3(x-1)} \rightarrow 2x + 2 = 3x - 3 \rightarrow x = 5$.

⚠ **Trap:** A 20% decrease means multiply by 0.80, not 0.20. And 'doubles every 4 years' is $2^{t/4}$, never 2t/4.

## 4. Function Transformations and Notation

**Recognize it:** g(x) = f(x − 3) + 2, or 'the graph of y = f(x) is shifted…', or a question that asks for f(a + 1) or f(g(2)).

**Method:**
**Inside** the parentheses moves the graph **horizontally** and in the **opposite** direction of the sign: f(x − 3) is 3 units RIGHT.
**Outside** moves it vertically in the expected direction: f(x) + 2 is 2 units UP.
−f(x) flips over the x-axis; f(−x) flips over the y-axis; 2f(x) stretches vertically.
**Composition:** f(g(2)) means find g(2) FIRST, then plug that number into f.

**Worked example:**
$f(x) = x^2 + 1$ and g(x) = 3x. Find f(g(2)).
g(2) = 6, then f(6) = 37. (Not g(f(2)) = g(5) = 15.)

⚠ **Trap:** The horizontal shift is backwards from what it looks like. f(x + 4) is LEFT 4. Say it out loud until it is automatic.

## 5. Circles: Completing the Square, Arcs, and Sectors

**Recognize it:** An equation like $x^2 + y^2 + 8x - 2y - 8 = 0$ and a question about the center or radius, or a circle diagram with a central angle asking for arc length or sector area.

**Method:**
**Standard form:** $(x - h)^2 + (y - k)^2 = r^2$, center $(h, k)$.
**Complete the square:** take half the x-coefficient, square it, add to both sides; repeat for y.
**Arc length** = (angle/360)·2πr.   **Sector area** = (angle/360)·$\pi r^2$.
Radians: degrees × π/180. A full circle is 2π radians.

**Worked example:**
$x^2 + y^2 + 8x - 2y - 8 = 0$
$(x^2 + 8x + 16) + (y^2 - 2y + 1) = 8 + 16 + 1$
$(x + 4)^2 + (y - 1)^2 = 25$ → center (−4, 1), radius 5.

⚠ **Trap:** The center signs are flipped: (x + 4) means h = −4. And the right side is $r^2$, so the radius is 5, not 25. Answer choices always include 25 as bait.

## 6. Right-Triangle Trig and Complementary Angles

**Recognize it:** sin, cos, or tan appears; or 'sin(x°) = cos(y°)'; or a right triangle where you are given one ratio and asked for another.

**Method:**
**SOH-CAH-TOA:** sin = opp/hyp, cos = adj/hyp, tan = opp/adj.
In any right triangle the two acute angles add to 90°, so **sin A = cos B** when A + B = 90.
Given one ratio, draw the triangle, use the Pythagorean theorem to find the third side, then read off the others.
Know the triples: 3-4-5, 5-12-13, 8-15-17, 7-24-25 (and their multiples).

**Worked example:**
In right triangle ABC (right angle at C), sin A = 7/25. Find tan B.
Opposite A = 7, hyp = 25 → adjacent = 24 (7-24-25 triple). Angle B is the other acute angle, so its opposite side is 24 and adjacent is 7: tan B = 24/7.

⚠ **Trap:** sin A and cos A live in the SAME triangle; do not draw two triangles. And when the problem says sin(x) = cos(y), the shortcut is simply x + y = 90.

## 7. Percent Change, Repeated Percents, and 'Percent of a Percent'

**Recognize it:** A price goes up 30% then down 30%; or 'x is 40% greater than y'; or a question asks for the percent change between two numbers.

**Method:**
**Percent change** = (new − old)/old × 100. The denominator is always the ORIGINAL.
**Repeated changes multiply:** up 30% then down 30% is 1.30 × 0.70 = 0.91 → a 9% decrease overall.
'x is 40% greater than y' → x = 1.4y. 'x is 40% of y' → x = 0.4y.

**Worked example:**
A shirt costs $80. It is marked up 25%, then the new price is discounted 20%. Final price?
80 × 1.25 × 0.80 = 80 × 1.00 = $80. No change at all.

⚠ **Trap:** Percents do not add and subtract. Up 25% then down 25% is NOT back to the start (it is 0.9375, a 6.25% loss).

## 8. Statistics Reasoning: Mean vs. Median, Standard Deviation, and Sampling

**Recognize it:** No calculation is really required. The question asks which statement 'must be true,' which data set has the larger standard deviation, or whether a survey result can be generalized.

**Method:**
**Outliers** pull the mean, not the median. A big high value → mean > median (skewed right).
**Standard deviation** measures spread. Values clustered near the mean → small SD. Values spread out → large SD. You never need to compute it on the SAT.
**Generalizing:** results apply only to the population the sample was randomly drawn from. A survey at a gym tells you about gym-goers, not all adults.
**Margin of error** shrinks with a larger sample; it does not change the estimate itself.

**Worked example:**
Set A: 20, 20, 21, 21, 22. Set B: 5, 15, 21, 27, 37. Both have mean 21. Which has the larger SD?
Set B: its values are far from 21, so its spread (SD) is larger.

⚠ **Trap:** 'Cannot be determined' is the right answer far less often than students pick it. If the sample is random from a population, you CAN generalize to that population.

## 9. Interpreting Linear Models in Context

**Recognize it:** An equation like $C = 35 + 0.15m$ with a story attached, and the question asks what a number or a variable 'represents' or 'means.'

**Method:**
$y = mx + b$. **m** (the slope) is the change in y for each ONE unit of x: 'per mile,' 'per hour,' 'each additional.'
**b** (the intercept) is the value of y when $x = 0$: the starting amount, flat fee, initial height.
If you are asked for the meaning of a solution, plug it in and translate the sentence.

**Worked example:**
A plumber charges $C = 60 + 45h$ for a job lasting h hours. What does 60 represent?
When $h = 0$ the cost is already 60, so 60 is the fixed charge for showing up, regardless of time.

> ⚠ **Trap:** The answer choices will offer 'the cost of one hour of work' for the intercept. That is the slope's job. Ask yourself: does this number change when x changes? Slope. Does it stay fixed? Intercept.

## 10. Rational and Radical Equations, Extraneous Solutions, and Absolute Value

**Recognize it:** A variable under a square root, a variable in a denominator, or |expression| in an inequality.

**Method:**
**Radical:** isolate the root, square both sides, solve, then **CHECK every answer** in the original. A square root is never negative.
**Rational:** note which values make a denominator zero BEFORE solving; those can never be answers.
**Absolute value:** $|A| < c \rightarrow -c < A < c$ (a sandwich). $|A| > c \rightarrow A < -c$ OR $A > c$ (two pieces).

**Worked example:**
$\sqrt{(x + 5)} = x - 1$. Square: $x + 5 = x^2 - 2x + 1 \rightarrow x^2 - 3x - 4 = 0 \rightarrow (x - 4)(x + 1) = 0$.
Check $x = 4$: $\sqrt{9} = 3$ and $4 - 1 = 3$ ✓. Check $x = -1$: $\sqrt{4} = 2$ but $-1 - 1 = -2$ ✗. Only $x = 4$.

> ⚠ **Trap:** Squaring creates fake solutions. One answer choice will be 'both 4 and −1.' Do not choose it.

## 11. Polynomials: Factors, Remainders, and Zeros

**Recognize it:** p(x) is a polynomial, you are told p(3) = 0 or given a table of values, and asked which expression 'must be a factor' or what the remainder is when dividing.

**Method:**
**Factor theorem:** (x − a) is a factor if and only if p(a) = 0.
**Remainder theorem:** the remainder when p(x) is divided by (x − a) is p(a).
A zero at x = 3 corresponds to the factor (x − 3), NOT (x + 3).
Reading a graph: x-intercepts are zeros; a zero where the graph just touches (does not cross) is a repeated factor.

**Worked example:**
$p(x) = x^3 - 2x^2 - 5x + 6$. Is (x − 3) a factor?
p(3) = 27 − 18 − 15 + 6 = 0. Yes. (No long division needed.)

> ⚠ **Trap:** Sign flip again: p(−2) = 0 means (x + 2) is the factor. Answer choices always offer both signs.

## 12. Geometry Scale Factors, Volume, and Density

**Recognize it:** Similar figures where one side is scaled; 'the radius is doubled, by what factor does the volume change'; or mass = density × volume word problems.

**Method:**
If lengths scale by k, **areas scale by $k^2$** and **volumes scale by $k^3$**.
Volume formulas are given in the reference sheet; density is not: **density = mass/volume**.
Convert units BEFORE computing (cm vs. m, minutes vs. hours). Write the units on every number.

**Worked example:**
Triangle ABC ~ triangle DEF with AB = 4 and DE = 10. Area of ABC is 12. Area of DEF?
k = 10/4 = 2.5, so area scales by $2.5^2$ = 6.25 → 12 × 6.25 = 75.

> ⚠ **Trap:** Doubling the radius of a sphere multiplies volume by 8, not 2. Students who scale by k instead of $k^2$ or $k^3$ will always find their wrong answer waiting in the choices.

## One-Page Formula & Fact Recap

| Topic | What to remember |
|---|---|
| Discriminant | $b^2 - 4ac$: >0 two, =0 one, <0 none |
| Vertex | x = −b/(2a); plug in for min/max value |
| Sum / product of roots | −b/a and c/a |
| Systems: no solution | coefficients proportional, constants not |
| Systems: infinite | one equation is a multiple of the other |
| Exponential | a(1 ± r)^(t/n) |
| Shifts | f(x − h) + k: right h, up k (inside is backwards) |
| Circle | $(x - h)^2 + (y - k)^2 = r^2$; right side is $r^2$, not r |
| Arc / sector | (θ/360)·2πr and (θ/360)·$\pi r^2$ |

| | |
|---|---|
| Trig | SOH-CAH-TOA; $\sin A = \cos(90° - A)$ |
| Percent change | (new − old)/old; repeated changes multiply |
| Mean vs median | outliers move the mean, not the median |
| Slope / intercept | per-unit rate / starting value ($x = 0$) |
| Radical eqs | square, solve, CHECK for extraneous |
| $\lvert A \rvert < c$ | $-c < A < c$; $\lvert A \rvert > c$ → two separate pieces |
| Factor theorem | $p(a) = 0 \Leftrightarrow (x - a)$ is a factor |
| Scaling | length $k$ → area $k^2$ → volume $k^3$ |
| Density | mass = density × volume (convert units first) |

**Test-day habit for a repeat taker:** on the hard module, before you compute anything, write the section number from this sheet (1–12) next to the question. Naming the type first is the single biggest predictor of getting it right.